Lecture Ch. 5

Donnerstag, 1. September 2022 10:15

5

$$\frac{\Delta Y_t}{10} = L_t - L_{t-1}$$

$$= (1 - UR_t) \cdot Pop_t - (1 - UR_{t-1}) Pop_{t-1}$$

$$= (1-UR_t)Pop_t - (1-UR_t)Pop_{t-1} + (1-UR_t)Pop_{t-1} - (1-UR_t)Pop_{t-1}$$

$$= (1-UR_t)(Pop_t - Pop_{t-1}) + Pop_{t-1}(1-UR_t-(1-UR_{t-1}))$$

$$= \Delta Pop_t - UR_t \Delta Pop_t - \Delta UR_t \, Pop_t$$

→ neg. relationship between ΔY_t and both UR_t and ΔUR_t !

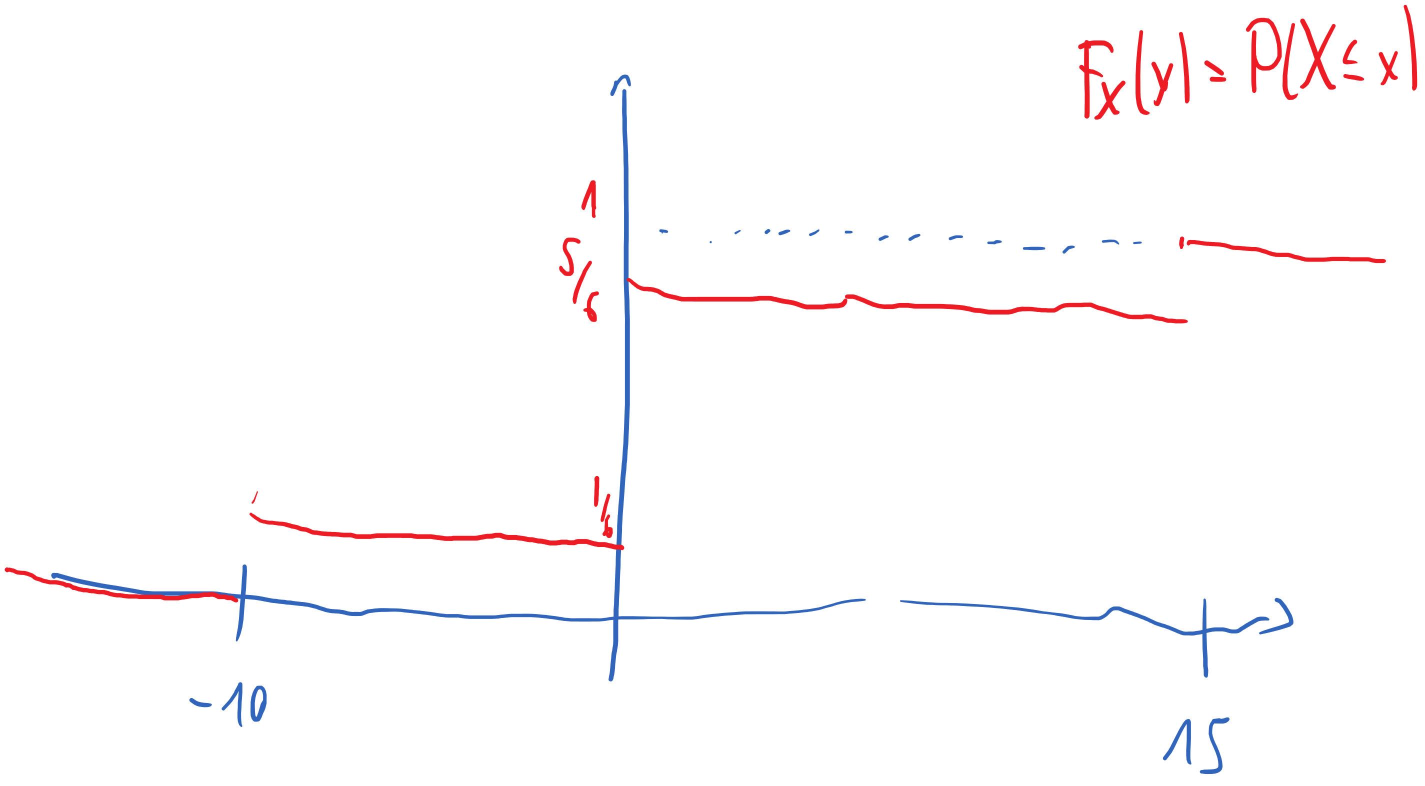
in ice cream / shark attacks: seasonality as the omitted factor.

9

X: $P(X=-10) = 1/6$

$P(X=0) = 4/6 = 2/3$

$P(X=15) = 1/6$



$$E(X) = \frac{1}{6} \cdot (-10) + \frac{4}{6} \cdot 0 + \frac{1}{6} \cdot 15$$

$$= \frac{15 - 10}{6} = 5/6$$

$$Var(X) = \mathbb{E}(X^2) - \mathbb{E}(X)^2$$

$$= 1/6 \, (-10)^2 + 4/6 \cdot 0^2 + 1/6 \cdot 15^2 - (5/6)^2$$

$$= 1/6 \, (100 + 225) - 25/36 \approx 53.5$$

$$sd(X) = \sqrt{Var(X)} \approx \sqrt{53.5} \approx 7.31$$

12

$$\| y - Xb \|_2 = \sqrt{(y - Xb) \cdot (y - Xb)}$$

$\rightarrow$ can equivalently optimize $(y - Xb)'(y - Xb)$

$$= y'y - y'Xb - (Xb)'y + (Xb)'(Xb)$$

$$= y'y - 2y'Xb + b'X'Xb$$

FOC for $\hat{\beta} = \arg\min_{b \in \mathbb{R}^2} \| y - Xb \|_2$:

$$0 = \frac{d}{db}\left(y'y - 2y'Xb + b'X'Xb \right)$$

$$\overset{\text{Ex. III.4}}{=} 0 - 2y'X + b'\left(X'X + \underbrace{(X'X)'}_{= X'(X')' = X'X} \right)$$

$$= -2y'X + 2b'X'X$$

Plug $\hat{\beta}$: $\Rightarrow$

$$\hat{\beta}' X'X = y'X$$

$$\Leftrightarrow \left(\hat{\beta}' X'X \right)' = (y'X)'$$

$$\Leftrightarrow \quad X'X\hat{\beta} = X'y$$

If $X'X$ is invertible: unique solution
$$\hat{\beta} = (X'X)^{-1} X'y.$$

Definiteness of $X'X$? For $v \in \mathbb{R}^2$:

$$v'X'Xv = (\underbrace{Xv}_{=:w})' \cdot Xv = w'w = \Sigma_i w_i^2$$

$$\geq 0$$

For $v \neq 0$, $v'X'Xv = 0$ if and only if $w = 0$, i.e. $X \cdot v = 0$,

or equivalently
$$\begin{pmatrix} 1 \\ \vdots \\ 1 \end{pmatrix} \cdot v_1 + \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} v_2 = 0$$

By $v \neq 0$, $v_2 \neq 0$, and therefore

$x_i = x_j \quad \forall i,j \in \{1, \dots, n\}$

$\rightarrow$ If X varies, this does not happen

$\Rightarrow$ $X'X$ is pos. definite and thus

invertible.

1S : error:

$$\hat{\beta} - \beta = (X'X)^{-1}X'y - \beta$$

$$= (X'X)^{-1}X'(X'\beta + e) - \beta$$

$$= \underbrace{(X'X)^{-1}X'X}_{I_2}\beta + (X'X)^{-1}X'e - \beta$$

$$= (X'X)^{-1}X'e \cdot \frac{n}{n}$$

$$= \left(\frac{1}{n} X'X\right)^{-1} \cdot \frac{1}{n} X'e$$