E600 Mathematics

Welcome and Chapter 0: Fundamentals of Mathematics

Martin Reinhard

August 24, 2022

Welcome to E600 Mathematics!

Overview

- What is E600 Mathematics?
  - Introductory module on mathematical background of economists
  - Course is not mandatory, no exam
  - more detail in a minute...
- Instructor: Martin Reinhard
  - PhD student at CDSE
  - Email: mareinhard@mail.uni-mannheim.de
  - no scheduled office hours, individual meetings can be arranged

- Main Course Objectives
  - Discuss and practice mathematical concepts central for economics
  - Get more familiar and comfortable with logic and notation
  - Know and be able to apply key theorems
  - Have a rough idea of why they are true (as "why" is sufficient for "how")
- Why care?
  - Mannheim has a strong quantitative orientation
  - Logical reasoning and mathematical methods are central to economics

  $\Rightarrow$ Make your life a bit easier in the years to come

1. Logistics and Organization

What: Content

0. Fundamentals of Mathematics (Logic, Functions, Sets)
1. Vector Spaces
   - Mathematical distance concepts, esp. norm-based distance
   - Set properties: bounded, open/closed, compact, etc.
2. Matrix Algebra
   - math with matrices: addition, multiplication, inversion
   - linear equation systems
3. Differential Calculus
   - Computing derivatives and integrals with vector functions
4. Optimization
   - Unconstrained and constrained problems in more than one variable
5. Econometrics
   - Introduction to probability theory (stochastics)
   - Techniques of constructing empirical models, basic estimators

1. Logistics and Organization

When?

- Overall structure
  - Classroom sessions: about 2/3 lecture, 1/3 exercise
  - Self-study: try 1h per 1h of class; review lectures + work on exercises
- Course structure (see also excel file on website)
  - Week 1 (Aug 22–28): online via Zoom
    - Wed-Fri, 9:15–10:45 and 11:00–12:30
  - Week 2 (Aug 29 – Sep 4): in person in L9, 1-2, room 004
    - Mon-Fri, 9:15–10:45 and 11:00–12:30
    - Every other day (Mon, Wed, Fri): afternoon session 13:30–15:00

1. Logistics and Organization

How? 1/2

- Web: `e600.uni-mannheim.de`
  - Full text for all chapters as Online Course
    - Interactive design: short questions, review quizzes
    - Focused on core competencies: few proofs, involved concepts left out
  - Additional practice problems for each chapter
- Course Material (available at `e600.uni-mannheim.de/downloads`)
  - Self-study and in-class exercise sheets, slides
  - Companion script: everything we do in class (and more) + all proofs
- Online Course & Script
  - *comprehensive* resources: meant as your MSc companion to consult for any issue related to mathematics
  - *self-sufficient* resources: consistent notation, rely only on known/previously introduced concepts

1. Logistics and Organization

How? 2/2

- Attend class
  - Class-based module "forces" you to spend time with the material
  - I can help you to read between the lines
  - Don't hesitate to **ask questions!** (also after class)
  - Learn how to come up with solutions in the exercise sessions
- Self-study
  - Online course + script, online exercises
  - Work through chapters/topics selectively & at your own pace
  - Work on self-study ("homework") problems
    - Afternoons are (mostly) left free: review + exercises
    - Expectation: understand all problems, solve the easier ones
    - No need to hand in anything
    - . . . but: math is a lot of "learning by doing" rather than "learning by listening", so you may learn most from your own work on exercises!

- E600 has **a lot of material**
  - many important concepts to cover in little time
  - not all is new; you need not know everything by the end of this class
- After this class, you should...
  - be less afraid of mathematical notation and formal arguments
  - have heard of the math. concepts central to economics and know where to look them up
  - everything else is a plus!

- In 5 weeks, you are sitting on a problem set for Microeconomics A.
- The exercise asks you: "Show that the budget constraint given above defines a bounded set."
- Because you followed this class well, you will think:
  - ✗ "okay, I know exactly how it works, this should be easy!"
  - ✓ "I know intuitively what a bounded set is, and why it's helpful if we can show that the budget set is bounded. Let me quickly
    - look up the definition of a bounded set, and
    - check how I can go about showing boundedness."

Chapter 0: Fundamentals of Mathematics

2. Fundamentals: Why Math?

aka "Why not a review of Micro/Macro/Econometrics?"

- Economists (among many other professions) put great emphasis on logical, rational reasoning – math is the purest form of this
- Economic investigations: mathematical or statistical models
- Math is actually less messy than the real world
  - No conceptual ambiguity: "fair tax system" vs. "Derivative of a given function f"? (existence?)
  - Full information: statements can be proved/disproved with certainty

  $\Rightarrow$ "lab conditions" to test logical reasoning
- Chapter 0: Fundamentals
  - Course assumes that the covered concepts are $\pm$ known
  - In class: very quick recap of contents + exercises

- Math = "language" very similar to normal English
  - Vocabulary: words + notation; sentence = "statement"
  - Important "sentences": quantifiers
    - Existential quantifier $\exists \ldots$, universal quantifier $\forall \ldots$
    - Example: $\forall n \in \mathbb{N} : n \geq 0$
  - Negation of statements: usually more direct when re-written
    - E.g.: $\neg(x \in A) \Leftrightarrow x \notin A$
    - Quantifiers: $\neg(\forall \ldots : S) \Leftrightarrow (\exists \ldots : \neg S)$ and vice versa (cf. exercises)
- Argument: statement about relationship of statements ("if ..., then")
  - English: "Any animal that is not a cat or a dog is a horse."
  - Math: $\forall a \in A : ((a \notin C \wedge a \notin D) \Rightarrow a \in H)$
  - Mathematical proof: establish non-obvious validity of argument

- Necessary and sufficient conditions for statement $S := (x \in \mathbb{N})$
  - sufficient: $x = 2$ (if ..., then S is true)
  - necessary: $x \in \mathbb{Z}$ (S is true only if ...)
  - equivalent: $(x \in \mathbb{Z} \wedge x \geq 0)$ (S is true if only if ...)
- Set = collection of distinct objects ("elements"...)
  - $\{1, 2, \pi\}$ vs. $\{1, 1, \pi\}$ vs. $\{n \in \mathbb{N} : n > 10\}$
  - Intervals: $I \subseteq \mathbb{R}$, write $(a, b) = \{x \in \mathbb{R} : a < x < b\}$
  - Operations (can be illustrated using circles)
    - Union $A \cup B$
    - Intersection $A \cap B$
    - Set difference $A \backslash B$

4. Fundamentals: Functions

- Usual notation for functions: $f : X \mapsto Y, x \mapsto y = f(x)$
  - X: domain, Y: codomain
  - Mapping rule "$x \mapsto y = f(x)$" relates each $x \in X$ to exactly one $y \in Y$
  - f is a function, "$f(x)$" is not!
  - Graph $G(f)$ of f (*relation* defining the function):

$$G(f) = \{(x, y) \in X \times Y : y = f(x)\} = \{(x, f(x)) : x \in X\}$$

- Key related concepts
  - Image $f[A]$ of $A \subseteq X$, preimage $f^{-1}[B]$ of $B \subseteq Y$
  - With $g : Y \mapsto Z$: composition $g \circ f : X \mapsto Z, x \mapsto g(f(x))$
  - Inverse function f^{-1}: $f^{-1} \circ f = f \circ f^{-1} = Id$ (existence: *bijectivity*)
    - Id: identity function for which $Id(x) = x$
    - Careful: inverse function at x, $f^{-1}(x)$ vs. preimage of $\{x\}$, $f^{-1}[\{x\}]$

5. Fundamentals: Further Concepts, Mathematical Proof

- Further concepts: univariate limits and continuity – ε/δ statements
- Mathematical proof
  - Recall: proofs establish non-obvious validity of a statement/fact
    - understanding them may improve your understanding of the fact
    - some are tedious (relative to the importance of the fact)
- So how do we proceed in terms of proofs?
  - The companion script gives all proofs in consistent notation
  - in class/exercises: easier proofs for the more central results
  - also: examples of methods of proof so that you have seen them once
    - Proof by induction
    - Equivalence proof in two steps
    - Contrapositive method of proof
    - ...