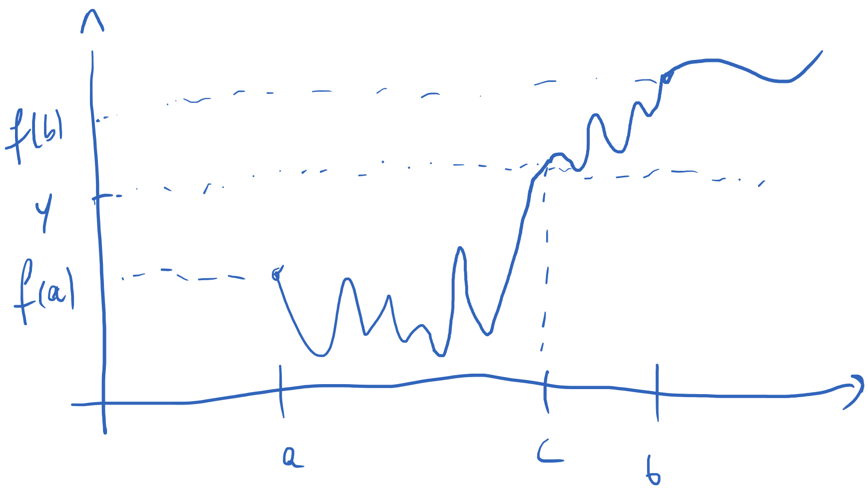
In-class 3

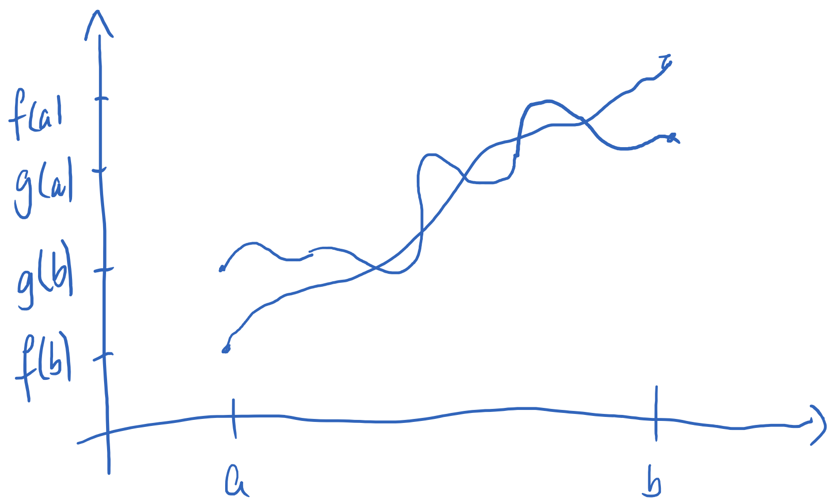
Dienstag, 31. August 2021 11:12

Problem 1



Theorem tells you that there is **at least** one intersection with y

a.)



Let $h(x) = g(x) - f(x)$. Then,

$h(a) = g(a) - f(a) \leq 0$

$h(b) = g(b) - f(b) \geq 0$

$\to$ by IVT, $\exists\, c \in [a,b]: h(c) = 0$, i.e.

$g(c) = f(c)$

b.) $f(x) = \sin(x) - \frac{3}{2}x$

Surjective: $\forall y \in \mathbb{R}: \exists x \in \mathbb{R}: f(x) = y$

Let $y \in \mathbb{R}$. Then,

$$\sin(x) - \frac{3}{2}x = y \iff \underbrace{\sin(x)}_{g(x)} = \underbrace{y + \frac{3}{2}x}_{h(x)}$$

for $x = \frac{2}{3}(1-y)$: $y + \frac{3}{2}x = y + (1-y) = +1 \geq \sin(x)$

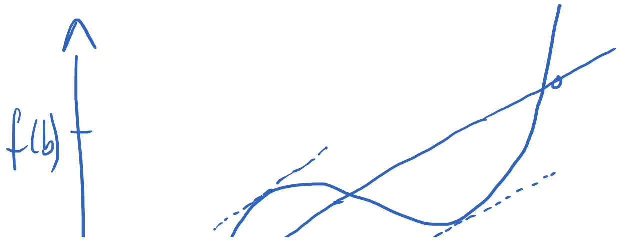
for $x = -\frac{2}{3}(1+y)$: $y + \frac{3}{2}x = y - (1+y) = -1 \leq \sin(x)$

$\to$ by a., $\exists\, x \in \left(-\frac{2}{3}(1+y), \frac{2}{3}(1-y)\right): y + \frac{3}{2}x = \sin(x)$

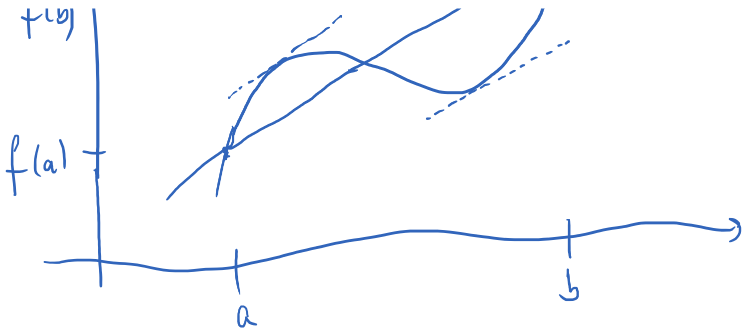
$\iff \sin(x) - \frac{3}{2}x = y$

=> f is surjective!

Problem 2



$\frac{f(b) - f(a)}{b-a}$ (b as the varying component)



Taylor-expansion of order 0 around a:

$$T_{a,0}(x) = f(a) \qquad \text{(approximation)}$$

$$f(x) = T_{a,0}(x) + \varepsilon_{a,0}(x)$$

$$= f(a) + f'(a+\lambda(x-a)) \cdot (x-a)$$

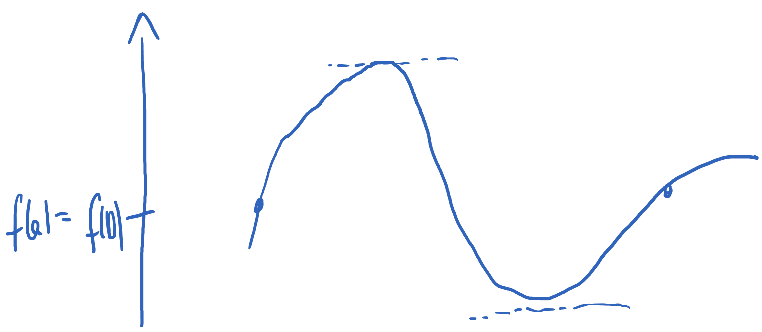
for some $\lambda \in (0,1)$

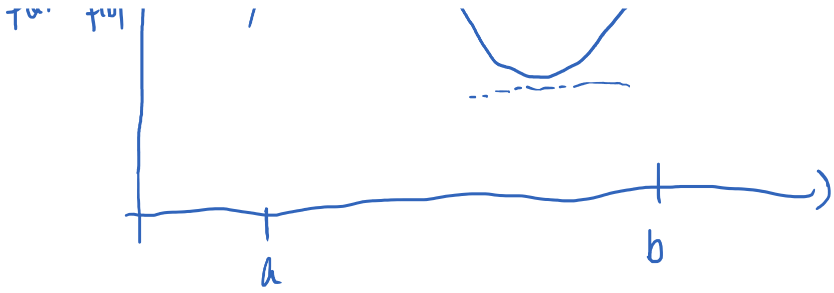
$$\Rightarrow f(b) = f(a) + f'(\underbrace{a+\lambda(b-a)}_{:= x_0 \in (a,b)})(b-a)$$

$$\Rightarrow f(b) - f(a) = f'(x_0)(b-a)$$

$$\Rightarrow f'(x_0) = \frac{f(b) - f(a)}{b-a}$$

✓





=> critical values exist on X if $f(x) = f(y)$ for some $x \neq y$, $x, y \in X$