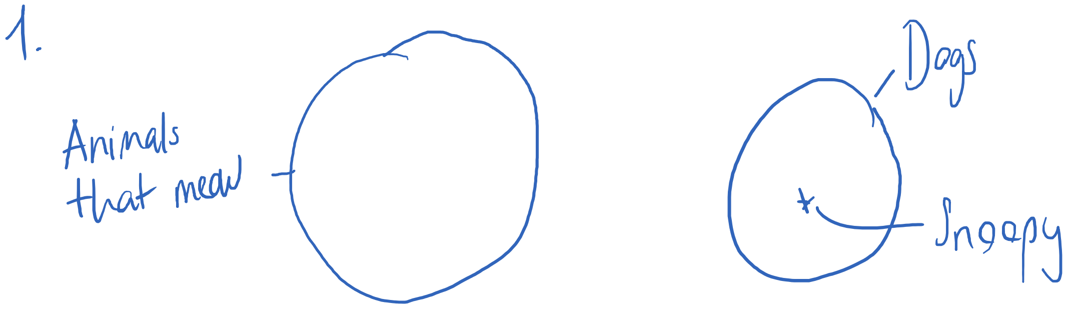
Solutions to problems not discussed in class

Dienstag, 24. August 2021 19:19

Homework 1

Exercise 1: Statements related to sets

1.



✓ valid

for b.: let

$M := \{\text{animals that meow}\}$

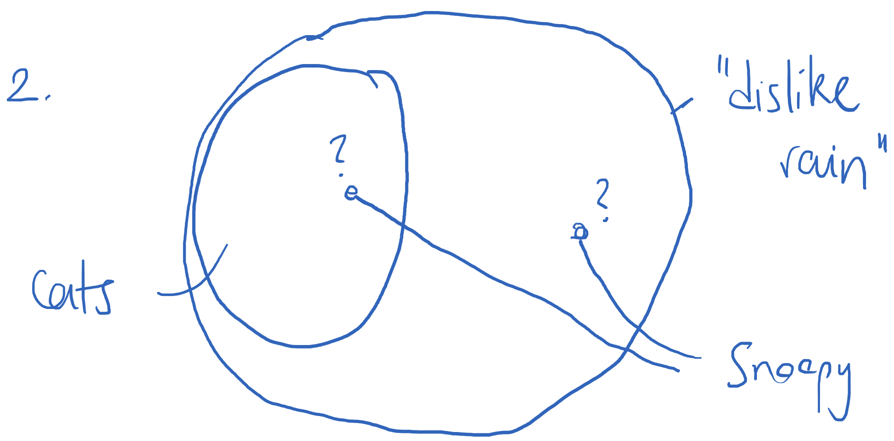
$D := \{\quad \text{— " —} \quad \text{are dogs}\}$

$s := \text{"Snoopy"}$

Then, we can write

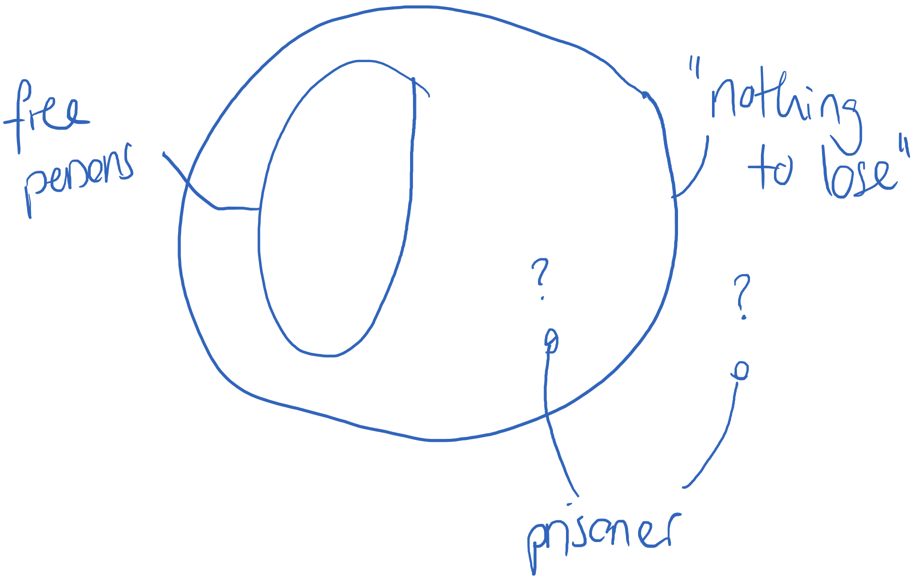
$$(D \subseteq M^c \wedge s \in D) \Rightarrow s \in M^c$$

2.



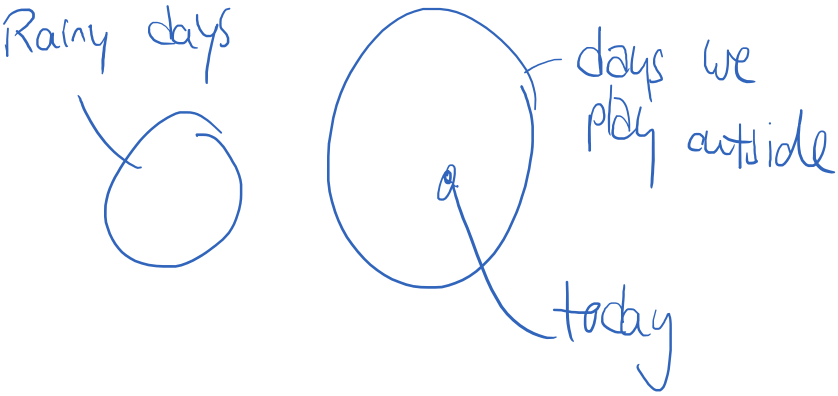
X not valid

3.



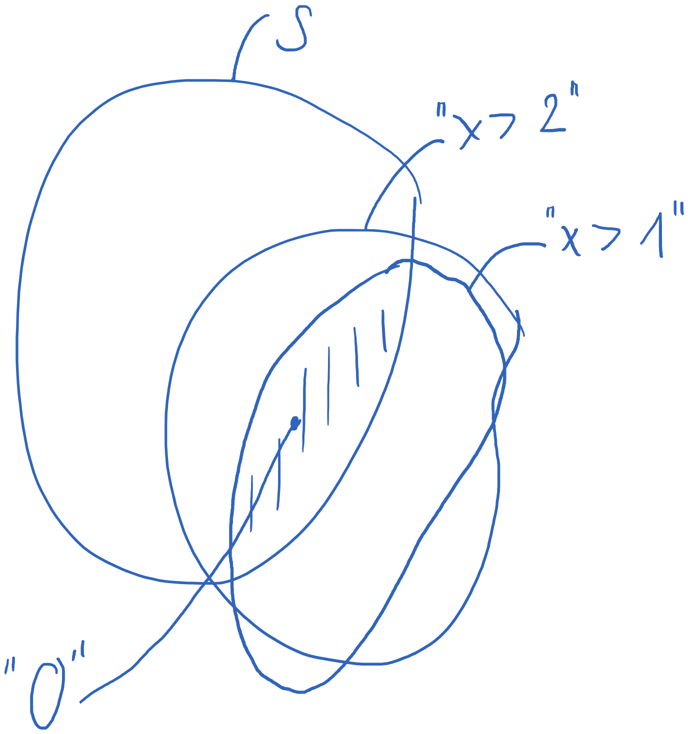
X not valid

4.



√ valid

5.



√ valid

for b.:

If we think of the relation > in the usual way, then the argument is not sound;

0>1 can never be true, violating the second premise. Note that the first premise may be possible depending on how S is defined; e.g. for S = $\mathbb{R} \setminus (-3,3)$ we know that within S, x>1 implies x>2.

Exercise 2b: Statements related to sets

1.: correct
2.: false; element sign the wrong way around ($B \ni 3$ would work)
3.: correct
4.: false: the set $\mathbb{N}$ of natural numbers contains numbers, not sets; therefore, the set A can not be contained in $\mathbb{N}$ (indeed: $A \subseteq \mathbb{N}$)

5.: false; the set on the RHS refers to all even numbers, not just those smaller than 10
6.: false
7.: correct
8.: false, sets do not contain duplicates
9.: false, the set {2,4,6,8,10} is not contained in A, which contains only numbers
10. and 11.: the two statements tell you "n is an uneven number" and "n<10"; they must BOTH hold to describe B, therefore 10 is wrong and 11 is right
12.: false; the intersection (i) does not contain 10, and (ii) contains uneven numbers. we could modify:

$A = ([2,10] \cap \mathbb{N}) \setminus B$, or w/o using B:

$A = [2,10] \cap \{2n : n \in \mathbb{N} \setminus \{0\}\}$

1. is true and does not need to be modified.

Exercise 3

b.
You should arrive at 10 for the first and -27 for the second scalar product.

The scalar product of a vector with itself obeys the "sum-of-squares" property, i.e. it squares all the entry in the vector and sums these squares up. Therefore, it can never be strictly negative.

c.

Here, you need to check the three properties of a metric for the concrete example given in the exercise.

(i) for non-negativity, the binary metric just takes values 0 and 1, so is never strictly negative. Further, it being equal to zero is, by definition, equivalent to the two vectors being equal: $d_B(x,y) = 0 \Leftrightarrow x = y$

(ii) for symmetry, x being (un)equal to y is exactly the same thing as y being (un)equal to x, thus $d_B(x,y) = d_B(y,x)$

(iii) for the triangle inequality, you pick arbitrary $x, y, z \in X$ and distinguish two cases.

If x = y, then $d_B(x,y) = 0 \leq d_B(x,z) + d_B(y,z)$

by non-negativity.

If instead $x \neq y$, then either $x \neq z$ or $y \neq z$ so that

$$d_B(x,z) + d_B(y,z) \geq 1 = d_B(x,y)$$

and thus in any case

$$d_B(x,y) \leq d_B(x,z) + d_B(y,z)$$