Review of Chapter 0 and In-class Exercises

Samstag, 21. August 2021 10:44

Welcome to the E600 Mathematics class of 2021!

Our key motivation, in a mathematical statement, is:

$$\forall h \in E600 : \left(t^{MSc} = H - h/2 \;\wedge\; f^{MSc} = F^{max} - e^{h}\right)$$

where t is time spent and f is frustration.

$$(x > 3 \;\vee\; x > 5)$$

A $x = 2$ X

B $x = 4$ ✓, as in standard english

C $x = 6$ ✓, "math." particularity

D $x = 100$ – " –

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$E \Rightarrow N$; $\neg(E \Rightarrow N)$ $\Leftrightarrow$ $\neg N \Rightarrow \neg E$

$\neg E \Leftarrow \neg N$

reconveine at 11:11

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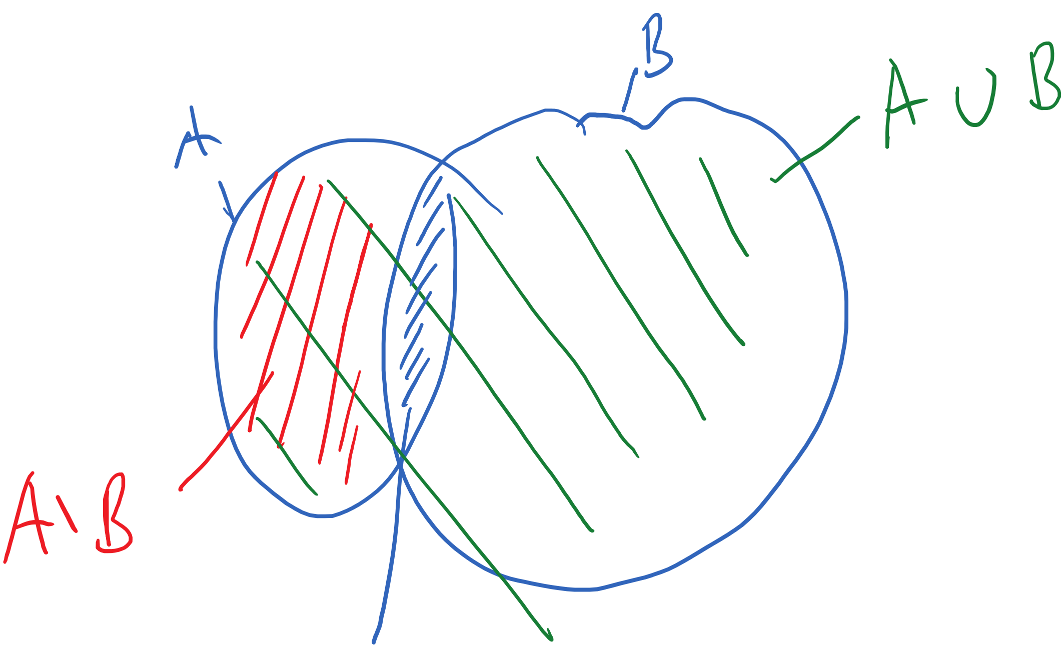
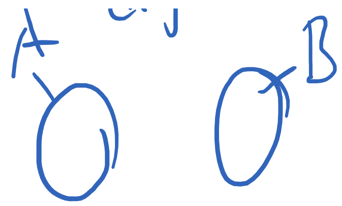


A disjoint B



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"$f(x) = \sin(x)$"

$$f(x,y) = \sin(x \cdot y)$$

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$$f[A] = f[[1,4]] = \{y \in \mathbb{R} : \exists(x \in [1,4] : f(x) = \sqrt{x} = y)\}$$

$$= [\sqrt{1}, \sqrt{4}] = [1,2] \subseteq Y = \mathbb{R}$$

$$f^{-1}[B] = f^{-1}[[0,3)] = [0^2, 3^2) = [0,9)$$

$$g \circ f(x) = g(f(x))$$

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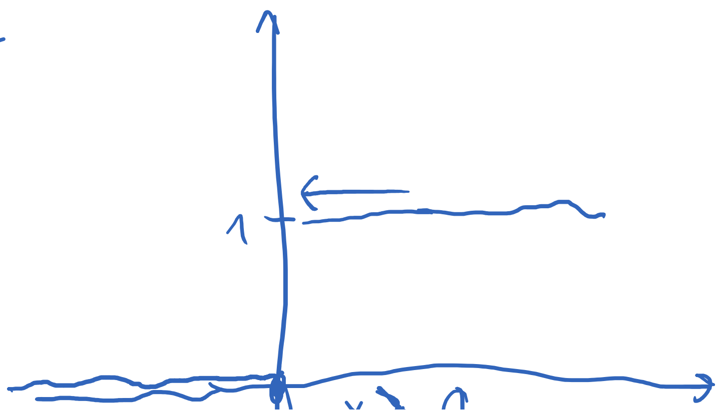

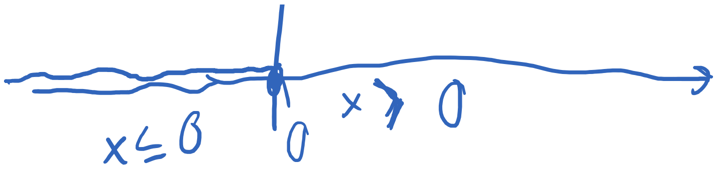

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$$\underbrace{\cos(n)}_{\in[-1,1]} + \underbrace{\mathbb{1}[n \text{ "even"}]}_{\in\{0,1\}} + 2 \le 4$$

$1 \le$

$$0 \le \frac{1}{n\left(\underbrace{\cos(n) + \mathbb{1}[n \text{ "even"}] + 2}_{\xrightarrow{n\to\infty} 0}\right)} \le \frac{1}{1\cdot n} = \frac{1}{n}$$

$0 \xrightarrow{n\to\infty} 0$, $\frac{1}{n} \xrightarrow{n\to\infty} 0$

In-class ex. 0

1a

1. $A \subseteq \{5\} \wedge \underline{\underline{A \supseteq \{5\}}}$ ✓ $\Leftrightarrow A = \{5\}$

1. $5 \in A$

2. $5 \in B \wedge 4 \notin B$

3.
$\forall n \in \mathbb{N} : (n \geq 0)$

$\nexists n \in \mathbb{N} : (n < 0)$

b.

1. $\neg(\exists n \in \mathbb{N} : n < 0)$

$\Leftrightarrow \forall n \in \mathbb{N} : (n \geq 0)$ ✓

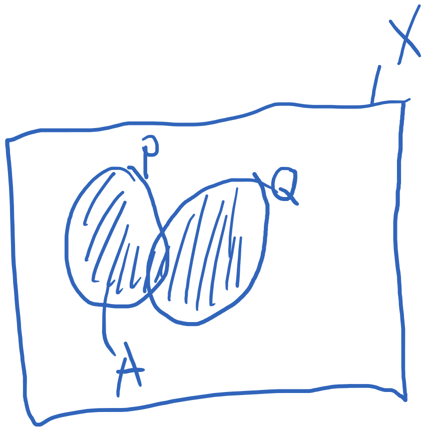
2. $\neg(\forall x \in \mathbb{R} : (x-1 > 0 \Rightarrow x > 0))$

$\Leftrightarrow \exists x \in \mathbb{R} : \begin{pmatrix} x-1 > 0 \wedge x \leq 0 \\ x-1 > 0 \Rightarrow x \leq 0 \end{pmatrix}$

$x - 1 > 0 \Leftrightarrow x > 1$

$\Rightarrow \neg(x \leq 0) = x > 0$

4. $\neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q$



$A^c = \{x \notin P \wedge x \in Q\}$
$= P^c \cap Q^c$

Problem 2

1. $|\emptyset| = \#$ elements contained in the empty set
$= 0$

2. $|\mathcal{P}\{\emptyset\}| = |\{\emptyset\}| = 1 = 2^0 = 2^{|\emptyset|}$
$= |\{ \{\} \}|$

3. $A = \{1, \pi\}$; $|\mathcal{P}\{A\}| = 4 = 2^2 = 2^{|A|}$

$\mathcal{P}\{A\} = \{\emptyset, \{1\}, \{\pi\}, \{1, \pi\}\}$

4. Indeed, it generally holds that $|\mathcal{P}(S)| = 2^{|S|}$

P. 3

Pr. 3

b. $\lim_{x \to 0} \frac{x}{e^x - 1} = \lim_{x \to 0} f(x)$

- If f is defined at $x = 0$ & continuous, compute $f(0)$

L'Hôpital's rule:

$$\lim_{x \to 0} \frac{x}{e^x - 1} = \lim_{x \to 0} \frac{\frac{d}{dx} x}{\frac{d}{dx}(e^x - 1)} = \lim_{x \to 0} \frac{1}{e^x}$$

$$= \frac{\lim_{x \to 0} 1}{\lim_{x \to 0} e^x} = \frac{1}{e^0} = 1.$$