

In-class Exercises for Chapter 0

Discussed in class on Monday, week 1

Topics: Logic, Notation, Sets, Functions and Limits

Problem 1: Notation and Logic

a.) Writing Formal Statements (online)

Write down the following verbal statements in formal notation.

Example. All real numbers are also rational numbers.

Answer: $\forall x \in \mathbb{R} : (x \in \mathbb{Q})$

1. The set A contains the number 5.
2. The set B contains the number 5 but not the number 4.
3. No natural number is strictly negative (that is, strictly smaller than zero).
4. If x is positive and y is negative, then (this implies that) the product $x \cdot y$ is negative.
5. At any multiple of π , the sin-function is equal to zero.
6. For any natural number and any integer, if the integer is positive, then their product is positive.

b.) Negation of Statements

Negate the following statements. Is the negation true (1.-3.)?

1. $\exists n \in \mathbb{N} : n < 0$.
2. $\forall x \in \mathbb{R} : (x - 1 > 0 \Rightarrow x > 0)$.
3. $\forall x \in \mathbb{R} : x \in \mathbb{N}$.
4. $P \vee Q$, where P and Q are arbitrary statements.

Problem 2: Set Theory

a.) Cardinality and Power Set

Remember that the cardinality of a set A , $|A|$, denotes the number of elements in a set.

1. What is the value of $|\emptyset|$?
2. What is the value of $|\{\emptyset\}|$?
3. If $A = \{1, \pi\}$, what is the value of $|\mathcal{P}(A)|$?
4. Express the value of $|\mathcal{P}(A)|$ as a function of $|A|$.

b.) Relationship of Set Operations (online)

Consider the relationship $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$. Is it true for arbitrary sets A , B and C ? Can you give a reasoning for why or why not, respectively? What about the analogous statement when we change intersections to unions and vice versa?

Problem 3: Functions

a.) A Derivative (online)

Take the derivative of $f : \mathbb{R} \mapsto \mathbb{R}, x \mapsto f(x) = \cos(x)/x^2$.

b.) A Limit (online)

Compute $\lim_{x \rightarrow 0} \frac{x}{e^x - 1}$. (Hint: L'Hôpital's rule)