E600 MATHEMATICS

Problem Set 0: Logic, Notation, Sets, Functions and Limits

Fall Semester 2019, course taught by: Martin Reinhard

Problem 1: Basic Logic

a.) Validity of Arguments

Are the following arguments valid (asserted relationship is always the implication)?

Nr.	Premise 1	Premise 2	Conclusion
1	All dogs do not meow	Snoopy is a dog	Snoopy does not meow
2	All cats dislike rain	Snoopy dislikes rain	Snoopy is a cat
3	A free person has nothing to lose	A prisoner is not a free person	A prisoner has something to lose
4	If it rains, we don't play outside	We play outside	It's not raining
5	For all $x \in S$, if $x > 1$, then $x > 2$	$0 \in S$ and $0 > 1$	$0 > 2$

Is 5. sound if $S \subseteq \mathbb{R}$?

b.) Arguments and Sets

1. Write down Nr. 1 of 1.a) as a set statement.

2. Draw Nr. 2 of 1.a.) using the "circle approach" to sets.

c.) Negation of Statements

Negate the following statements. Is the negation true (1.-3.)?

1. $\exists n \in \mathbb{N} : n < 0$.

2. $\forall x \in \mathbb{R} : (x - 1 > 0 \Rightarrow x > 0)$.

3. $\forall x \in \mathbb{R} : x \in \mathbb{N}$.

4. $P \vee Q$, where P and Q are arbitrary statements.

Problem 2: Set theory

a) Cardinality and Power Set

Remember that the cardinalty of a set A, $|A|$, denotes the number of elements in a set.

1. What is the value of $|\emptyset|$?
2. What is the value of $|\{\emptyset\}|$?
3. If $A = \{1, \pi\}$, what is the value of $|\mathcal{P}(A)|$?
4. Express the value of $|\mathcal{P}(A)|$ as a function of $|A|$.

b) Statements related to Sets

Let $A = \{2, 4, 6, 8, 10\}$ and $B = \{1, 3, 5, 7, 9\}$. Which of the following statements are true?

1. $2 \in A$
2. $3 \ni B$
3. $4 \notin B$
4. $A \in \mathbb{N}$
5. $A = \{2n : n \in \mathbb{N} \backslash \{0\}\}$ (set of even numbers)
6. $A \cup B = \mathbb{N}$
7. $A \cup B \subset \mathbb{N}$
8. $A = \{2, 4, 6, 8, 10, 2, 4, 6, 8, 10\}$
9. $A = \{2, 4, 6, 8, 10, \{2, 4, 6, 8, 10\}\}$
10. $B = \{n \in \mathbb{N} : ((\exists m \in \mathbb{N} : n = 2m + 1) \vee n < 10)\}$
11. $B = \{n \in \mathbb{N} : ((\exists m \in \mathbb{N} : n = 2m + 1) \wedge n < 10)\}$
12. $A = [2, 10) \cap \mathbb{N}$

c) Evaluation of Statements using Proofs

Let $A, B, C \subset X$. Prove or disprove the following statements:

1. $B \cup (\emptyset \cap A) = B$
2. $((A \cap B) \cup (A \cup B^c))^c = B$
3. De Morgan's Law: $(\bigcup_{i \in I} A_i)^c = \bigcap_{i \in I} A_i^c$

Problem 3: Functions

a) Codomain and Range

Give an example for a function f for which the codomain is not equal to the range of f.

b) Image of a Set under a Function

Let $f : X \to Y$ be a function, and let $A \subset X$. If we say that y is an element of $f[A]$, i.e. $y \in f[A]$ what exactly do we know about y?

A. $f(y) \in A$.
B. $f^{-1}(y) \in A$.
C. $y \in X$.
D. For some $x \in A$, it holds that $f(x) = y$.
E. $y \in A$.

c) Preimage of a Set under a Function

Let $f : X \to Y$ be a function, and let $B \subset Y$. If we say that x is an element of $f^{-1}[B]$, i.e. $x \in f^{-1}[B]$, what exactly do we know about f and x?

A. $f(x) \in B$
B. $\exists y \in B : x = f^{-1}(y)$
C. $x \in B$
D. $f(x) = B$
E. f is invertible.
F. $f^{-1}(B) = x$

d) Derivative using the Appropriate Rule

Calculate $f'(x)$ for $f : \mathbb{R} \mapsto \mathbb{R}, x \mapsto \sin((2x + 4)^2)$.